MODE DISPERSION IN GRADED-INDEX OPTICAL FIBER WITH NEAR PARABOLIC-INDEX PROFILES

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ABSTRACT

The propagation constants and the group delay time of the guided modes in the graded-index fiber with near parabolic-index profiles can be determined in almost analytic form within the WKBJ approximation. The minimum mode dispersion in this fiber can be attained by correcting the fourth order term to the square law profile after the refractive-index of the cladding can be adjusted.

Introduction

The refractive-index distribution in the cross section of a multimode optical fiber has as well as the homogeneous cladding an important influence on the propagation characteristics of the guided modes. For the uncladded graded-index fiber with near parabolic-index profiles, the optimum index profile in the sense of the minimum mode dispersion has been evaluated theoretically.[1,2,3] Theoretical analyses for the cladding effect on the mode dispersion can be done in the special case such as parabolic-index fiber,[4,5] except for the numerical method.[6]

We analyze the cladded graded-index fiber with the refractive-index profiles of which are expressible in terms of the truncated power series of the form : $n(r)=n_1(1-(gr)^2+\alpha(gr)^4-\beta(gr)^6)^{1/2}$, where n_1, g, α, and β are constants, using the WKBJ method. The fiber parameters for the minimum mode dispersion are evaluated within the WKBJ approximation. The mode dispersion in the near cutoff frequencies can be equalized by constructing the sharp index step between the core and the cladding. The minimum mode dispersion can be attained by correcting the fourth order term to the parabolic-index profile with holding the cladding situation obtained above.

Formulation of problem and WKBJ solution

Let us consider the circular symmetric rod with gradient core and uniform cladding extending to infinity. We use the cylindrical coordinate system (r,θ,z). The refractive-index distribution of the fiber is described by

$$n(r) = \begin{cases} n_1(1-h(r))^{1/2}, & r \leqq a \\ n_1(1-2\Delta)^{1/2}, & r \geqq a, \quad h(r)=(gr)^2-\alpha(gr)^4+\beta(gr)^6 \end{cases} \tag{1}$$

where Δ is a positive constant and 'a' denotes a core radius. The time dependence $\exp(j\omega t)$ is understood and the modes propagate along the axial direction according to $\exp(-j\beta_{n,\mu}z)$. It should be mentioned that we use the number μ instead of the meridional mode number m for the uncladded fiber: $\mu=m+\Delta\mu\,(m=0,1,2,\ldots)$.

Under the well-known approximation, $1\gg h(r)$; $1\gg\Delta$,[7] the problem for determining the guided modes can be reduced to solving the differential equation of the form, in the core region,

$$\Phi_i''(r)+(1/r)\Phi_i'(r)+[k^2(\chi-h(r))-\nu^2/r^2]\Phi_i(r)=0, \quad i=1,2$$

$$\text{with } \beta_{n,\mu}=k(1-\chi)^{1/2},\ k=k_0n_1,\ \nu=\begin{cases}1-n, & \text{for } i=1\\ 1+n, & \text{for } i=2\end{cases} \tag{2}$$

where the prime denotes the derivative with respect to r and k_0 is the wavenumber in vacuum. For $n\neq0$, the suffix '1' describes the HE-mode and '2' the EH-mode.[7] For n=0, we have the TE-mode or the TM-mode. The electromagnetic field for the guided modes in the core region can be represented in terms of the functions $\Phi_i(r)$ and $U_i(r)$ defined by

$$U_i(r)=-(1/k)[\Phi_i'(r)+(\nu/r)\Phi_i(r)], \quad i=1,2. \tag{3}$$

Since the field in the cladding can be expressed by the modified Bessel function $K_n(vr)$ with $v=k(2\Delta-\chi)^{1/2}$ and its derivative, matching the boundary condition at r=a leads to the characteristic equations of the form

$$\Phi_i'(a)-(vK_{|\nu|}'(va)/K_{|\nu|}(va))\Phi_i(a)=0, \quad i=1,2. \tag{4}$$

Now let us express the solution of (2) in the concrete form. By using a damped solution $\Phi_i^1(r)$ and a growing solution $\Phi_i^2(r)$ with respect to r of (2), the WKBJ solution near the core boundary can be represented in the form

$$\Phi_i(r)=C[\Phi_i^1(r)-\tan\left(\int_{r_1}^{r_2}P(k,r)^{1/2}dr-\pi/2\right)\Phi_i^2(r)], \quad i=1,2$$

$$\text{with } P(k,r)=k^2(\chi-h(r))-\nu^2/r^2 \tag{5}$$

where r_1 and r_2 are two zero points of $P(k,r)$ and called the turning points, and C is a constant. From (4) and (5), we obtain the resonance equations

$$\int_{r_1}^{r_2} P(k,r)^{1/2}dr = (\mu+1/2)\pi, \quad m=0,1,2,\ldots, \quad i=1,2 \tag{6}$$

where

$$\Delta\mu=(1/\pi)\tan^{-1}[(\Phi_i^{1'}(a)+Q\Phi_i^1(a))/(\Phi_i^{2'}(a)+Q\Phi_i^2(a))], \quad i=1,2$$

$$\text{with } Q = -vK_{|\nu|}'(va)/K_{|\nu|}(va). \tag{7}$$

For the refractive-index distribution h(r) in (1), From (6) we can get the propagation constants in the following form:

$$\beta_{n,\mu}=k[1-sg/k+\alpha(3s^2/8-\nu^2/2)(g/k)^2 +\{(17\alpha^2/64-5\beta/16)s^3-3(3\alpha^2/4-\beta)\nu^2s/4\}(g/k)^3]^{1/2}, \quad i=1,2 \tag{8}$$

where $s=4\mu+2(|\nu|+1)$ (see the Appendix). The turning points are also expressed by the power series of g/k. Substituting the WKBJ solution into right hand side of (7), we have the transcendental equations with respect to $\Delta\mu$. Solving these numerically, from (8) we obtain the propagation constants of the guided modes. In order to evaluate the mode dispersion, let us calculate the group delay time per unit length defined by

$$t = (n_1/c)d\beta_{n,\mu}/dk$$

where c is the light velocity in vacuum. From (8) we get the group delay time in the form

$$t=(n_1/c)[1-\{2kd(\Delta\mu)/dk\}(g/k) +(1/8)\{s(s-8kd(\Delta\mu)/dk)(1-3\alpha/2)+2\alpha\nu^2\}(g/k)^2+\ldots], \quad i=1,2 \tag{9}$$

It should be noted that for the uncladded fiber ($\Delta\mu=0$) (9) coincides with that obtained by the ray theoretical method.[2] Using (7), we can also estimate $d(\Delta\mu)/dk$ and $\Delta\mu$ numerically. Substituting these into (9), we can calculate the mode dispersion of the guided modes in the graded-index fiber with the refractive-index (1).

Numerical results and discussion

To examine the cladding effect on the mode dispersion, we use the parameters a_κ and κ which are related to each other through $h(a_\kappa)=(2+\kappa)\Delta(\kappa\geq 0)$. Then the V parameter is defined in usual manner such that $V=ka_0(2\Delta)^{1/2}$. The a_κ's denote core radius. In order to check the validity of the WKBJ solution, we calculate the values of $\Delta\mu$ and compare these with those obtained by the rigorous method.[4] Fig.1 shows that both solutions are in good agreement with each other. As shown in Fig.2, the mode dispersion at the near cutoff frequencies can be equalized by constructing the sharp index step between the core and the cladding. Fig.3 shows that we can specify the parameter α for minimizing the mode dispersion, after the sharp index step can be adjusted. In fact, the fourth order correction to the parabolic-index profile leads to the minimum mode dispersion for the parameter α with $0.65<\alpha<0.75$ under $\kappa=2$ (see Fig.4). From Fig.5, we can see that the relative group delay time for $\alpha=0.70$ is about three times smaller than for $\alpha=0$, that is, the parabolic-index fiber.

Conclusion

The mode dispersion of the guided modes in the cladded optical fiber with near parabolic-index core can be evaluated by the WKBJ method. It turns out that the considerable improvement can be attained by adding the fourth order correction term to the parabolic-index fiber.

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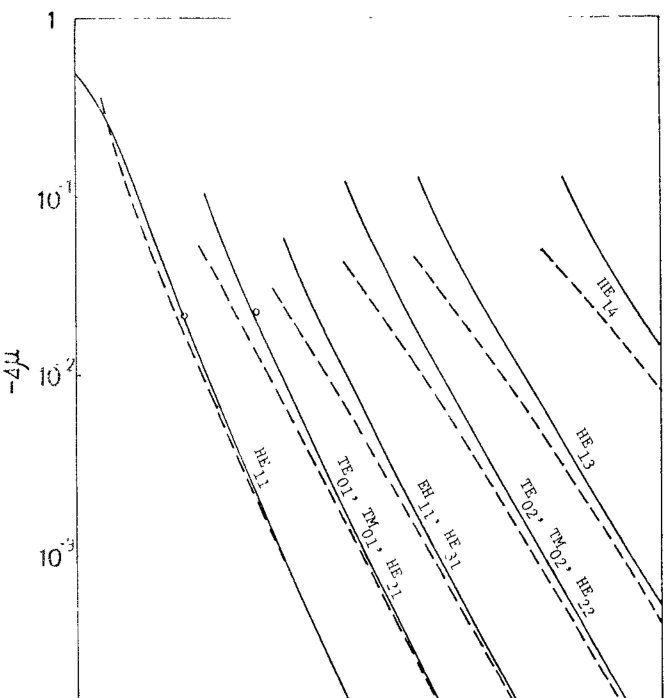

Fig.1. The values of $-\Delta\mu$ for the guided modes in the parabolic-index fiber. The dashed curves indicate the first-order asymptotic solutions and the circles show the exact ones.[4] $\alpha = \beta= 0,\Delta = 0.01,\kappa = 0$.

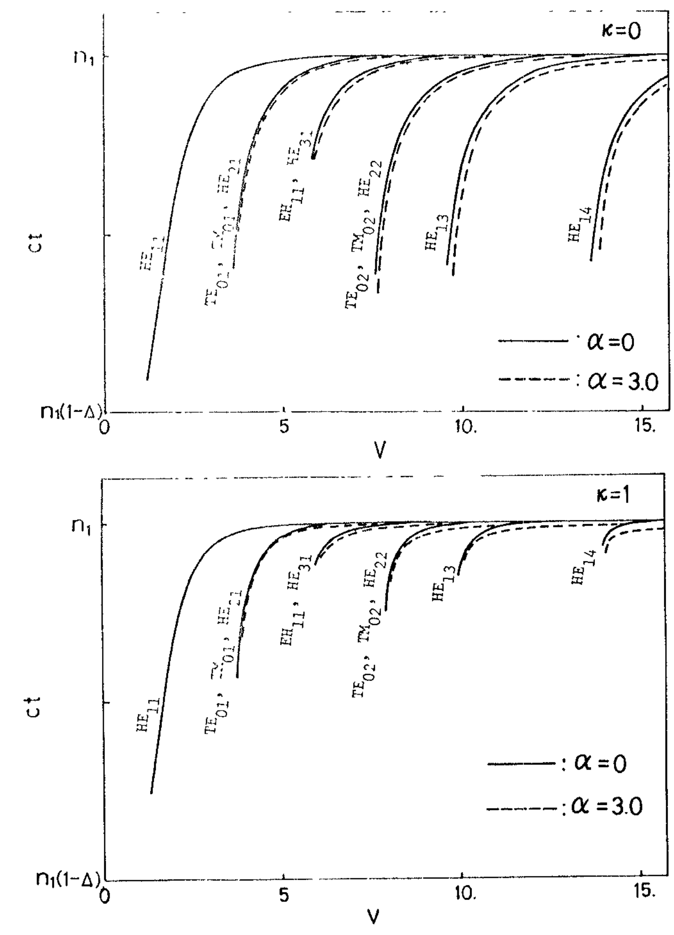

Fig.2. The cladding effects for group delay time ct. $\beta = 0$, $\Delta = 0.01$.

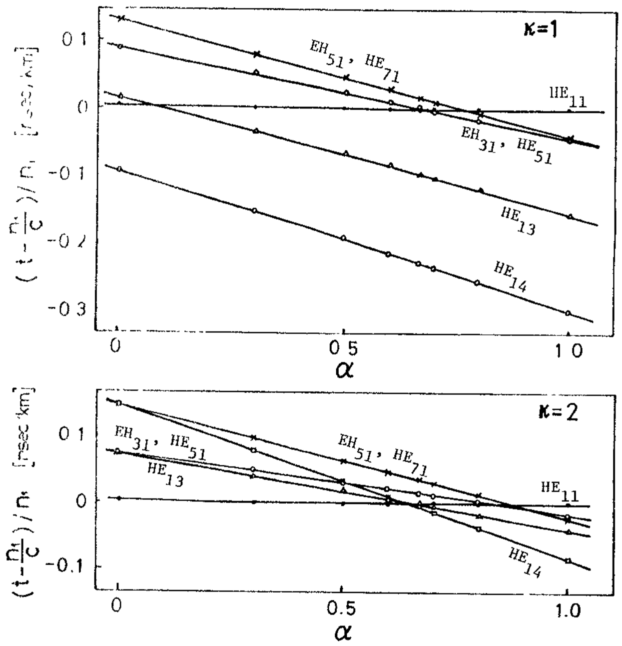

Fig.3. The relative delay time considered cladding effect versus fourth order correction. V=15.0, β= 0, Δ=0.01.

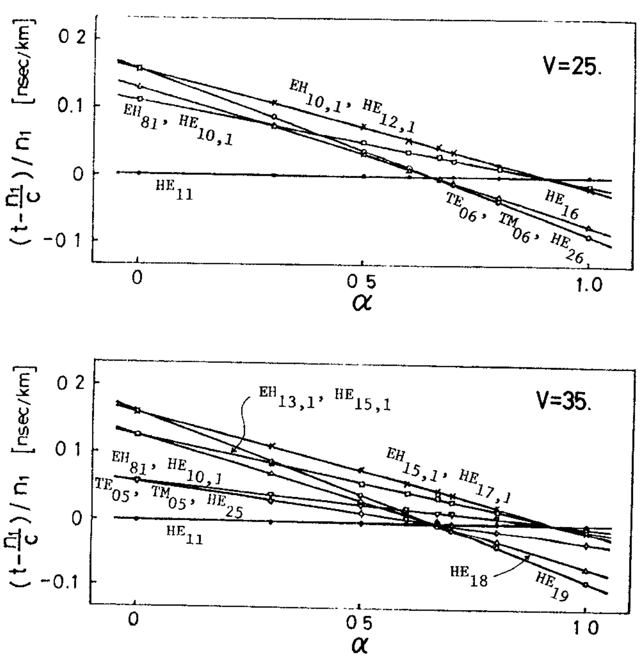

Fig.4. The optimum parameter α for the minimum relative delay time. β = 0,Δ = 0.01, κ = 2.

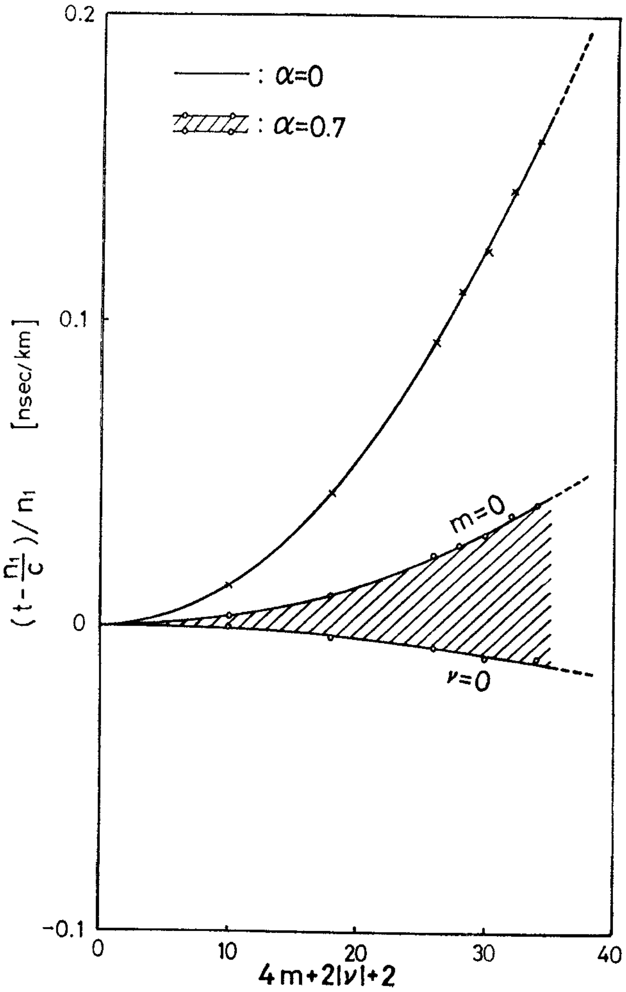

Fig.5. The relative delay time for α=0.70 and α=0 versus 4m+2(|ν|+1). The dashed curves show the values for the uncladded fibers. V=35.0,β = 0,κ = 2.

Appendix

Making the change of variable $(gr)^2=R$ and substituting h(r) in (1) into (6), we have

$$P(k,r)=k^2(\delta-R)(R-\gamma)[1-\alpha(\delta+\gamma)+\beta((\delta+\gamma)^2-\delta\gamma) -(\alpha-\beta(\delta+\gamma))R+\beta R^2]/R,\ \gamma=(gr_1)^2,\ \delta=(gr_2)^2. \quad (A.1)$$

In the course of the derivation of (A.1), we also have

$$\beta_{n,\mu}=k[1-(\delta+\gamma)+\alpha((\delta+\gamma)^2-\delta\gamma)-\beta(\delta+\gamma)((\delta+\gamma)^2-2\delta\gamma)]^{1/2} \quad (A.2)$$

and

$$\nu^2(g/k)^2=\delta\gamma[1-\alpha(\delta+\gamma)+\beta((\delta+\gamma)^2-\delta\gamma)]. \quad (A.3)$$

Expanding the quantity in the brackets of (A.1) into Taylor series about R=0, the left hand side of (6) can be evaluated in closed form. Then from (6), we have

$$(\mu+1/2)(g/k)=(1/2)[1-\alpha(\delta+\gamma)/2-(\alpha^2/8-\beta/2)(\delta+\gamma)^2+\beta\delta\gamma/2] \times((\delta+\gamma)/2-(\delta\gamma)^{1/2})-(1/2)[\alpha/16+(5\alpha^2/128 -3\beta/32)(\delta+\gamma)]((\delta+\gamma)^2-4\delta\gamma). \quad (A.4)$$

Considering $1\gg g/k$, from (A.3) and (A.4) we have $1\gg\delta,\gamma$. By using (A.3) and (A.4), $\delta+\gamma$ and $(\delta\gamma)^{1/2}$ can be determined in the power series form with respect to g/k. Substituting these into (A.2), we obtain (8).